PDEs & Randomness school/conference

1st-7th July 2024

Abstracts and Course Descriptions

Abstracts

Pr.Ezzinbi Khalil *Dynamics and attractiveness of oscillatory solutions for some monotone differential inclusions: applications to parabolic and hyperbolic systems*

We study the existence of compact almost automorphic weak solutions for differential in- clusions governed by a maximal monotone operator A and the forcing term f is compact almost automorphic. We prove that the existence of a uniformly continuous weak solution on having a relatively compact range over implies the existence of a compact almost automorphic weak solution. For that goal, we use Amerio's principle. We also prove the existence, uniqueness, and global attractivity of a compact almost automorphic weak solution where A is strongly maximal monotone. For illustration, some applications are provided for parabolic and hyper-bolic equations.

Pr. Juraj Foldes *Local well posedness for cubic Schrodinger equation with random initial conditions*

During the talk, we will discuss the local solutions of the super-critical cubic Schrödinger equation (NLS) on the whole space with general differential operator. Although such a problem is known to be ill-posed, we show that the random initial data yield almost sure local well-posedness. Using estimates in directional spaces, we improve and extend known results for the standard Schrödinger equation in various directions: higher dimensions, more general operators, weaker regularity assumptions on the initial conditions. In particular, we show that in 3D, the classical cubic NLS is stochastically, locally well-posed for any initial data with regularity in H^ε for any $\varepsilon > 0$, compared to the known results $\varepsilon > 1/6$. The proofs are based on precise estimates in frequency space using various tools from Harmonic analysis. This is a joint project with Jean-Baptise Casteras (Lisbon University) and Gennady Uraltsev (University of Virginia, University of Arkansas).

Pr. Xueying Yu *Some unique continuation results for Schrödinger equations*

This talk focuses on a fundamental concept in the field of partial differential equations — unique continuation principles. Such a principle describes the propagation of the zeros of solutions to PDEs. Specifically, it answers the question: what condition is required to guarantee that if a solution to a PDE vanishes on a certain subset of the spatial domain, then it must also vanish on a larger subset of the domain. Motivated by Hardy's uncertainty principle, Escauriaza, Kenig, Ponce, and Vega were able to show in a series of papers that if a linear Schrödinger solution decays sufficiently fast at two different times, the solution must be trivial. In this talk, we will discuss unique continuation properties of solutions to higher-order Schrödinger equations and variable-coefficient Schrödinger equations, and extend the classical Escauriaza-Kenig-Ponce-Vega type of result to these models. This is based on joint works with S. Federico-Z. Li, and Z. Lee.

Dr. Rhoss Likibi Pellat *On the Feynman-Kac formula and discrete-functional FBSDEs: Beyond the Lipschtz framework*

In this talk, we will address the long-standing issue of representing adapted solutions to quadratic BSDEs as functional of a diffusion process (via a system of partial differential equations solution), whenever the terminal value is permitted to be a discontinuous discrete functional of the same diffusion process. Furthermore, we will show that the latter Feynman-Kac formula remains valid in the case where the terminal value and drift coefficient of the diffusion process are both outside the Lipschitz-continuous framework, by utilizing some quite sophisticated purely probabilistic methods for studying the regularity of the gradient solution of semilinear PDEs, . The findings reported above have practical implications, particularly when considering the numerical aspect of quadratic BSDEs with a singular diffusion process. Furthermore, these findings could be used to investigate the propagation of singularities in time for quadratic BSDE with non-Lipschitz terminal values.

Dr. Sidy Moctar Djitte *On Lions' formula for Reproducing Kernal Hilbert Space of s-harmonic functions*

A reproducing kernel Hilbert space (RKHS) is a Hilbert space H of functions in which pointwise evaluations are continuous linear functionals. It then follows from Riesz' theorem that each of these functionals can be represented as an inner product with an element of this Hilbert space, i.e, for any x, there exists K_x in H such that for any f in H we have $f(x) =< K_x, f >_H$. The reproducing kernel of H is defined for any pair x and y by $K(x, y) :=< K_x, K_y >_H$. This notion was first introduced in 1907 by Stanislaw Zaremba for boundary value problems for harmonic and biharmonic functions, and simultaneously by James Mercer in the theory of integral equations, before to be more systematically tackled by Nachman Aronszajn and Stefan Bergman. These spaces have various applications in complex analysis, harmonic analysis, quantum mechanics and statistical learning theory.

One of the last theorems by Jacques-Louis Lions was about a formula for the reproducing kernel $K(,)$ of some function spaces associated with second order elliptic boundary value problems. An interesting observation by M. Englis et al in the work "On the formula of Jacques-Louis Lions for reproducing kernels of harmonic and other functions" is the resemblance of Lions' formula with the Hadamard variation formula for the Green function of the classical Laplacian. In the same work, it is said that the authors "are of the opinion that the connection between Hadamard's varation formula and the reproducing kernel just indicated seems to be an isolated phenomenon peculiar to the second order case."

In this talk, we first establish Hadamard variational formula for the Green function of fractional Laplace operators of s in $(0, 1)$ and then showed that actually, a similar connection between Lions formula for RKHS and Hadamard's varation formula occurs as well for fractional Laplace operators, answering in a negative way to the question raised by M. Englis et al. The talk is based on the joint work "A few representation formula for solutions of fractional laplace equations" with Franck Sueur (University of Bordeaux).

Pr. Mickael Latocca *Non-invariance of Gaussian Measures under the 2d Euler Flow*

In this talk I will explain why we expect that Gaussian measures are generally not expected to be invariant (and even not quasi-invariant) under the flow ot the 2D Euler equations apart from the one very specific case: the Gibbs meausure which has very low regularity. Then I will show how simple arguments leads to proving that high reularity gaussian measures are not invariant under the 2d Euler flow.

Dr. Rosati Tommaso *The Allen-Cahn equation with weakly critical initial datum*

Inspired by questions concerning the evolution of phase fields, we study the Allen-Cahn equation in dimension 2 with white noise initial datum. In a weak coupling regime, where the nonlinearity is damped in relation to the smoothing of the initial condition, we prove Gaussian fluctuations. The effective variance that appears can be described as the solution to an ODE. Our proof builds on a Wild expansion of the solution, which is controlled through precise combinatorial estimates. Joint works with Simon Gabriel, Martin Hairer, Khoa Lê and Nikos Zygouras.

Pr. Alex Blumenthal *TBA*

Courses

Dr. Nicola Kistler *A crash course on mean field spin glasses*

An introductory course in the statistical mechanics of disordered systems. In these lectures, I will try to give a comprehensive overview of the alas huge field of disordered systems called mean field spin glasses.We will introduce the prototypical Sherrington-Kirkpatrick model, and then briefly recall the so-called Parisi solution from theoretical physics, in particular the wonderfully mysterious replica computations. We will then discuss an alternative treatment, always from theoretical physics, the one advocated by Thouless, Anderson and Palmer (TAP) and Plefka. We will then move to the state of the art of the mathematically rigorous treatment, with focus on the contributions by Guerra and Talagrand, and discuss the link between the two theories, Parisi and TAP, which goes through Derrida's models, with applications to the log-correlated class. Time permitting, I will conclude with some pointers to current, ongoing research on the algorithmic treatment of the TAP-Plefka equations; these are random fixed point equations in very large dimensions, the analysis of which is of relevance in the study of "all things complex networks", such as machine learning, AI, and much, much more.

Dr. Florian Bechtold *The variational approach for SPDEs and pathwise extensions*

TBA

Pr. Olivier Menoukeu Pamen *Existence, uniqueness and path by path uniqueness of stochastic wave equation*

In this course, we discuss well posedness of stochastic wave equation when the vector field is not smooth. We study an SDE driven by a two parameter Brownian motion with rough drift. The approach relies on a local time-space representation of Brownian sheet and a type of law of the iterated logarithm for the Brownian sheet. We also discuss the Malliavin smoothness of the solution to the equation.